ACCESS: CONTEMPORARY ISSUES IN EDUCATION
1982, VOL. 1, NO. 2, 73–87

The unity of knowledge

C. W. Evers and J. C. Walker

Department of Education, University of Sydney, NSW, Australia

ABSTRACT
Knowledge is better viewed as a seamless web than as a partitioned set. We attempt to show this by developing an internal critique of Hirst's theory that there exists a certain number of logically distinct forms of knowledge. In the course of this critique we produce some very general results which tell against any form of epistemic fragmentation, whether based on logical, semantical, or epistemological grounds. We urge, finally and briefly, the credentials of a materialist view which avoids the problems of fragmentation, preserves the unity of knowledge, and accounts for the errors of the kind of view we are attacking.

> What harm in getting knowledge even from a sot, a pot, a fool, a mitten, or even an old slipper?
>
> (Rabelais, Gargantua and Pantagruel, 3.)

Knowledge is better viewed as a seamless web than as a partitioned set. We attempt to show this by developing an internal critique of Hirst's theory that there exists a certain number of logically distinct forms of knowledge. In the course of this critique we produce some very general results which tell against any form of epistemic fragmentation, whether based on logical, semantical, or epistemological grounds. We urge, finally and briefly, the credentials of a materialist view which avoids the problems of fragmentation, preserves the unity of knowledge, and accounts for the errors of the kind of view we are attacking.

The paper is divided into five sections:

1. Partitions
2. Semantics of Logical Relations
3. Derivability
4. Tests against Experience
5. Materialism

The tenor of each section of our overall argument is that the demands made by a theory that implies that knowledge can be partitioned outrun the resources provided by a good epistemology.

1. Partitions

Our strategy in this section is to show that Hirst's proposed partitioning of knowledge into distinct forms in any non-trivial sense, requires that he subscribe to a much stronger thesis, which we call the H-thesis.

The H-thesis states that there exists a relation R, defined on a set S, such that R partitions S into disjoint subsets where

i. Sis some subset of K, where K is the set of all knowledge claims, and
ii. the disjoint subsets are forms of knowledge.

Naturally, for this rather austere statement to be even comparable with Hirst's views about knowledge we need to give an account of certain key terms like 'disjoint', 'K', 'R', and 'S' that Hirst would either find acceptable, or that he would be obliged to accept. And to do this we need to examine in some detail a cluster of related terms Hirst uses to describe the basis on which knowledge is segmented.

One such term is 'knowledge' itself, or in our terminology, the elements of K. What are they? It seems to us that for Hirst the things that are partitioned into forms of knowledge are propositions. In "Forms of Knowledge Revisited" Hirst gives an unambiguously propositional view of knowledge, claiming that terms like 'philosophy', 'mathematics and logic', 'physical sciences I and so on "are to be understood as being strictly labels for different classes of true propositions".[1] Similarly, to the charge that his approach may be too propositional, Hirst in "Realms of Meaning and Forms of Knowledge" replies

> … those forms of meaning or intelligibility which are not themselves propositions must involve the use of concepts, and they in turn necessitate the existence of propositions and truth criteria of some kind.[2]

However, if Hirst's commitment to a propositional view of knowledge is relatively straightforward (though subject to occasional fuzziness - e.g. describing forms as "the complex ways of understanding experience"[3]), interpreting what he means by 'proposition' is not.

Notwithstanding Hirst's careful discussion[4] concerning propositions as meanings of words, phrases and sentences, we distinguish two important views. Meanings or propositions have sometimes been regarded as some kind of subtle entity, as an abstract or intentional or propositional object named by sentences. On this view for x, y, ε S and R a relation between propositions, in the closed sentence (x) (y) xRy, x and y are propositional variables whose values are propositions and whose appropriate substitutions[5] are therefore names of propositions. Although such an account sits ill with Hirst's jaundiced view of certain metaphysical doctrines underpinning what he calls the Greek notion of education, his failure to systematically distinguish between contexts of use and mention[6] is, we shall later suggest, of a piece with precisely this view.

A second account of propositions, and one with which Hirst would appear to agree, is that a proposition is what is expressed by a (declarative) sentence[7], the word 'expressed' in this context having no ontological import. With no further elaboration this is vague, but as it does permit sentences to represent, (in some strong sense that might even include identity), items of knowledge, it will be no part of our plan to sharpen it up. Instead, because propositions in this sense are at least isomorphic to sentences, we can, without loss of generality treat sentences as items of knowledge. On this view Hirst's forms of knowledge thesis is construed as a set of statements (i.e. sentences) about the fabric of sentences comprising human knowledge. To avoid certain complications to do with self-reference, we need to distinguish between an object language {OL) and a metalanguage (ML). The OL is the object of our study and contains all the sentences constituting knowledge claims. The ML contains all the sentences we use to express theses about OL sentences. In terms of the H-thesis, for x, y ε S, R stands for the name of some ML relation between OL sentences, and in the closed ML sentence (x) (y) xRy, x and y are variables with OL sentences as values and names (formed by quotation) of OL sentences as appropriate substitutions. Although we shall have principal recourse to formulations of arguments which treat of sentences as elements of S, in a primarily epistemological critique such as this, not too much hinges on this decision[8], and we shall occasionally offer reformulations conforming to the idiom demanded in countenancing propositional objects.

Does Hirst think all knowledge can be partitioned into one or other of his forms? In "Liberal Education and the Nature of Knowledge" he expresses his doubts. He suggests that

> .... the dividing lines that can be drawn between different disciplines by means of the four suggested distinguishing marks are neither clear enough nor sufficient for demarcating the whole of modern knowledge as we know it.[9]

We can further interpret the H-thesis in a manner consistent with this limitation by restricting S in clause (i) to just those items of knowledge that can, without difficulty, be assigned to a particular form. Stubborn cases and troublesome counter-examples might likewise be profitably removed from S if it will help in s ta ting the forms of knowledge thesis without further undue and ambiguous qualification. Although this may seem arbitrary, there is a limit. For presumably for Hirst there must be some items of knowledge that can be assigned in a clear and unambiguous way to different forms, otherwise his claims become, on his own account, implausible. And whatever other problems his. position encounters, it ought not be thrown into question by the unruly behaviour of a few recalcitrant elements of K.

Still, in spite of these restrictions on S, is there any reason for doubting that Hirst intends S to be partitioned into disjoint subsets, that is, subsets which have no elements in common? It would appear there is, for after having distinguished a particular sense of 'mutual irreducibility' for forms[10], Hirst adds

> (I)t was no part of the thesis … that the forms of knowledge are totally independent of each other, sharing no concepts or logical rules. That the forms have been interrelated has been stressed from the start.[11]

But a relation R, partitions S into disjoint subsets if and only if R is an equivalence relation.[12] And R is an equivalence relation if and only if, for x, y, z ε S (a) R is reflexive, xRx; (b) R is symmetric, xRy Ɔ yRx: and (c) R is transitive, xRy & yRz Ɔ xRz. So either 'mutual irreducibility means the same as ' disjoint', and 'not being totally independent' fails as a relevant. qualification, or there is some other sense of 'mutual irreducibility that is compatible with the partitioned sets sharing some elements.

Unfortunately, the latter alternative leads to a collapse of partitions. To see this, suppose some relation, Ri, partitions S into 'mutually irreducible' subsets P, Q, T, … , where P and Q are interrelated such that x, Y ε P and y, z ε Q. Then xRiy and yRiz. But either Ri is transitive or it is not. If it is, then xRiy & yRiz Ɔ xRiz, thus causing the distinction between P and Q to collapse since all the elements of P can be related to all the elements of Q by funnelling the relation through the common element y. If, on the other hand, Ri is not transitive, it cannot define a partition on S.

It is important to realize that there is no middle way here. If Ri is not transitive it cannot define a partition on S. If Ri is transitive and there is an element common to the two subsets, the distinction between the subsets collapses. It would appear then, that for there to be any point to the claim that forms of knowledge are 'mutually irreducible', Ri must be an equivalence relation partitioning S into disjoint subsets. So when Hirst speaks of the forms being 'not totally independent' or 'interrelated' he cannot mean that they share elements, at least not in this sense of sharing. Perhaps when Hirst suggests that the elements of subsets formed by partitioning S by some relation are interrelated across subsets he means that they are interrelated under some other relation. A relation R, can partition S into disjoint forms of knowledge without excluding the possibility of another relation, say, "...is presumed by..." being defined on Sand being satisfied by pairs of elements with each relatum drawn from a different form. If this is so then there is nothing problematical about forms of knowledge being 'mutually irreducible' yet 'interrelated' provided of course the partitioning relation

R, does not include or imply the 'interrelating' relation. However, while this clears up Hirst's account of the forms' being both 'independent' and 'interrelated', it renders implausible a corresponding analysis of ' totally independent'. For if 'independent' fails to rule out interrelationships, then presumably 'totally independent' by way of contrast would. Thus, we would have to say a relation partitions S into 'totally independent' disjoint subsets if no other relation can be defined on S. But this is not a real option, as equivalence relations on Scan be defined arbitrarily.

Unfortunately either we accept something like this as an account of the 'independent/totally independent' distinction with Hirst's qualification ' not totally independent' being trivially true and hence directed against a strawman objection, or we see the distinction as corresponding to the 'non-disjoint/ disjoint' distinction with its implied collapse of partitions for merely 'independent' subsets. As Hirst wants to say there are distinct forms of knowledge 'not totally independent' we reject the latter proposal and accept the former, reading terms like 'independent', 'mutually irreducible', 'logically. distinct' and so on, as alternatives for 'disjoint'. An ironically interesting corollary of this, though obviously one we shall waive, is that however knowledge is partitioned, it cannot be on the basis of certain "concepts or logical rules" as these are precisely the examples used to illustrate the interrelatedness of knowledge.

How then is knowledge to be partitioned? So far we have argued that Hirst's forms of knowledge thesis must satisfy certain minimum conditions to be a claim about segmenting knowledge at all. These are the conditions captured by our statement of the H-thesis. To prove that some knowledge claims can be partitioned into forms we therefore need to show there exists some (non-trivial) equivalence relation R, defined on these knowledge claims, S, such that the resulting partition of disjoint subsets is identifiable as a partition of S into forms of knowledge. Furthermore, to avoid the problem of defining R being translated into the problem of specifying criteria of identification, we accept, but only provisionally, as an ostensive account of forms, those (sometimes traditional) groupings of knowledge named by Hirst's terms 'formal logic and mathematics ', 'physical sciences', 'ethics', 'aesthetics' and so on. We can presume supposed paradigm elements admitted to S thus serve as privileged material for the construction of a relation exemplifying fundamental logical or epistemological cleavages among these initially roughly delineated knowledge groupings: It must be stressed that this provisional acceptance of divisions in knowledge is purely a methodological device to place some rough restrictions on the acceptability of particular relations. Whether these divisions are "...necessarily distinct or an accident of academic history and administrative convenience..."[13] is, of course, something still to be decided. This means that our intuitive feelings about differences in knowledge have no explanatory value, as it is precisely their accuracy that is at issue. Thus, to take a limiting case, we cannot permit the equivalence relation '… belongs to the same form as...' to function in an explanation of the basis on which knowledge is partitioned, as our understanding of the term 'form' in this context is no more intelligible or epistemically secure than our original intuitive notions currently in need of detailed theoretical underpinning.

Obviously what we require is some independent[14] specification of an R preferably in terms of purely logical notions, that will partition S into what we are prepared to recognise as intuitively acceptable divisions in knowledge.

2. Semantics of Logical Relations

Let us consider a number of candidates for logical relations first. To avoid begging any of the semantical issues, the best strategy would appear to be to begin with those logical relations most clearly understood; for our purposes the truth functional conditional '$\supset$' and the truth functional biconditional ' $\equiv$ '. But even to start here is already to invite some dispute, for on one significant widely held view in the philosophy of logic, (and one we happen to hold), '$\supset$' and ' $\equiv$ ' are not relations at all, but sentence connectives.[15] And if our elements of S are sentences, the difference is vital, being ultimately the difference between use and mention, or more specifically for us, OL and ML. Connectives function as conjunctions, standing between sentences, to form other (compound) sentences. For example,

grass is green Ɔ snow is white.

But the conditional and biconditional construed as relations function as verbs standing between names or mentionings of sentences, so forming other sentences. Since we have used the symbols 'Ɔ' and ' ≡ ' as OL expressions for the material conditional and the material bi conditional, it would be wiser to use other symbols to express the analogous metalinguistic truth functional relations of material implication and material equivalence; perhaps ' → ' and ' ↔ ' respectively. The ML sentence corresponding to our above OL example would then be

'grass is green' → 'snow is white'

Provided these distinctions are kept in mind, there is no reason to mistakenly countenance subtle entities such as propositional objects. But regrettably, as Hirst appears to (i) conflate use and mention and (ii) treat connectives as relations, this is precisely the mistake his analyses invite. Consider his claim: "What teaching implies is merely the intention to bring about learning".[16] As the expressions 'teaching' and 'the intention to bring about learning' do not occur within quotation marks, we assume they are used rather than mentioned. This suggests that 'implies' names some kind of (non truth functional) connective rather than a relation. But 'implies' in a context like this is exactly the sort of expression Hirst would call a logical relation. To have it both ways we must therefore assume 'implies' doubles as a connective between expressions and a relation between objects named by expressions. This confuses meaning with reference. The ontological commitment of this latter condition becomes explicit when we recast its claims in prenix normal form of the canonical notation of the first order predicate calculus thus: (∃x) (∃y) xRy, where R is the relational sense of ' Ɔ '.

Turning now to the metalinguistic two place semantical predicates '→' and ' H ', do they satisfy the H-thesis? Unfortunately no, for though '→' is a relation, it is not an equivalence relation, and while '↔ ' is, it is also truth functional and so merely partitions the elements of S into two subsets; sentences that are true and sentences that are false. But because S is a set of knowledge claims we can regard the set of false sentences as empty. Material equivalence as an equivalence relation therefore fails to partition S at all.

Clearly what Hirst needs is a semantically more robust logical relation; one that takes into account meanings and not merely truth values: Two candidates that were designed to do precisely this Job are Lewis's modal logic analogues of the material conditional and biconditional, namely, strict implication, '→ ', and strict equivalence ' ≣ '.[17] However, Lewis also confused use and mention, with the result that '→' and ' ≣ ' came out as sentence connectives, not relations (except between propositional objects). It would be better if they were called 'strict conditional' and 'strict bi conditional' and we reserve the expressions 'strict implication' and 'strict equivalence' for corresponding verbs flanked by quoted OL sentences appearing in ML. Let us do this, using the symbols '⇒ ' and '⇔' as names for the metalinguistic relations of strict implication and strict equivalence. Then, to revert to a revised version of an earlier example, we might want to say in a spirit sympathetic to Hirst

'x is teaching' ⇒ 'x in tends to bring about learning', where '⇒' would be understood as the relation' ... follows logically from…', and would obtain because 'intends to bring about learning' is part of the meaning of 'teaching'. In general '⇒' is said to hold between mentioned expressions when the meaning of the consequent is contained or included in the meaning of the antecedent.

Similarly, but this time for strict equivalence, we might want to say

'x is a bachelor' ⇔ 'x is an unmarried adult male'.

Here '⇔' is said to hold because the mentioned expressions are synonymous.

Assuming for the moment we are prepared to countenance meaning analysis in the required sense,[18] are these relations of any help to Hirst? Sadly, we can rule out '⇒' straight-away, as, like its material counterpart '→ ', it is not an equivalence relation. But strict equivalence is different, being an equivalence relation that partitions the elements of S into subsets of synonymous elements. For example, if S is a set of sentences with 'Hirst is a bachelor' as a member, then under ' ⇔ ', 'Hirst is a bachelor' would belong to a subset made up of all its synonyms. The case with propositional objects partitioned under the relational sense of ' ≡ ' is slightly different since there being some difference in meaning is often taken to be a criterion of individuation. But in this case we merely have a set of subsets each of which contains only one proposition. Obviously strict equivalence, though an improvement on material equivalence is too fine grained to correspond tb a partition of knowledge into forms.

If we are to continue to look for an appropriate logical relation , what we appear to require is one that is stronger than ' ↔ ' but weaker than '⇔', that takes into account more than just truth values but invokes less than outright synonymy. Where should we look? If we used sameness of meaning to specify a relation that grouped elements into subsets of synonyms, perhaps we should use some much weaker sense of 'sameness' to specify a relation that groups knowledge cl aims in to forms. Here the problem is to spell out this weaker sense of 'sameness' without invoking the locution 'form of knowledge' or its circle of cognates.

It seems to us that there is no way of providing an account of this relation, as a logical relation without appealing to the very notions the relation was supposed to explicate. To see why this is so, consider the set S, of OL knowledge claims. Presumably, OL mathematical statements, (to choose, arbitrarily, one example), are about mathematical objects like numbers, sets, functions and so on rather than about the objects of religious discourse (e.g. God, heaven, divine foreknowledge, etc.) or the objects of moral, aesthetic or philosophical discourse. Let us suppose furthermore, that we do in fact have a well defined (by whatever means) grouping of all the OL mathematical statements: Then to each of these sentences x, we can prefix the sentence operator M, standing for 'It is an item of mathematical knowledge that…' to form the corresponding true sentences Mx. So far so good. The set of expressions of mathematical truth Mx is as distinct and well defined as the set of mathematical truths. The trouble begins when we examine how Mx behaves in inferences. Consider, for example, the following moves:

(1) M3 > 2
(2) 2 = the number of coins in my pocket a t time t.
(3) M3 > the number of coins in my pocket at time t.

Unfortunately, the expression immediately to the right of M in (3) is not an item of mathematical knowledge. It presumably falls into the class of empirical truths discovered by the methods of science. The reason this occurs is because Mx is a referentially opaque construction.[19] This means that truth is not always preserved when singular terms to the right of Mare replaced by co-referring singular terms.

To see why inferences of the above sort are sometimes invalid, and to explore some of the standard moves in the literature for improving matters, we need to seek a bit of general perspective on the problem. We can begin by noting that (1) and (3) are particular instantiations of the quantification scheme

(4) (∃x) M (3 > x).

If the singular term '2' is substituted for x in (4) we have (1). Similarly, if the singular term 'the number of coins in my pocket at time t' is substituted for x in (4) we have (3). M(3 > x) comes out true under the former substitution, false under the latter. The reason this precipitates a crisis is because on the standard objectual interpretation of quantification, it is the values of x, the objects over which x ranges, that are relevant to the truth value of expressions with quantifiers, not the kinds of singular terms substituted for x. Since on this view how objects are referred to is strictly irrelevant, quantification into opaque contexts where truth depends on the mode of reference, is unintelligible.

If we want to keep standard quantification theory and the option of redescribing so called mathematical objects (e.g. numbers) then we should give up the operator M, that is, we should give up the task of trying to demarcate a special set of statements called 'mathematical truths'.

A response to this line of argument is to challenge the objectual interpretation of quantification. One such non-objectual interpretation which takes into account relevant non-referential differences between singular terms is substitutional quantification. Here (4) comes out true if and only if M(3 > x) is true for some expression substituted for x.[20] In this case '2' or '1 + 1' are suitable expressions, whereas 'the number of coins in my pocket at time t' is not. The reason why the argument from (1) to (3) fails is because (2) is a false premise. This is because on a substitutional reading of (2) the identity sign is construed as substitutional identity. As the terms flanking this sign are only coreferential and not substitutionally identical (2) is false. Thus, in order to sort out valid from invalid inferences involving M(3 > x), we need to be able to identify the set of appropriate truth preserving substitutions for x in (4).

As we have assumed (for the sake of argument) that we have these truth preserving substitutions at hand, recourse to substitutional quantification permits a coherent rendering of the Mx construction. This deft move is, of course, not without cost, and in the case of Hirst's epistemology the cost is particularly high. For substitutional quantification abstracts from reference altogether, and so the idea that mathematical, religious or scientific statements, for example, are about (in some sense} mathematical, religious or scientific reality, simply drops out. Moreover, if we now remove the epistemological assumption that we already possess a well-defined grouping of all the OL mathematical statements, the appeal to substitutional quantification as a basis for interpreting the role of M in patterns of inference now becomes circular, as it is parasitic upon a prior account of the conditions under which Mx constructions are true. What is needed is an independent account of why some expressions can be substitution instances of quantificational schemata involving M, and others cannot.

One approach is suggested by Follesdal's analysis of objectual quantification into referentially opaque constructions involving causal and epistemic operators.[21] Since we are now dealing with objectual quantification, it is the values of x that must be limited rather than the substitutions for x. Follesdal tries to achieve this by restricting the stock of singular terms by admitting only certain descriptions as genuine. This is achieved in the case of our earlier example by requiring genuine singular terms to satisfy

(5) (x) (2 = x Ɔ M(3 > x)).

In (2) the singular term to the right of the identity sign fails this condition. According to Follesdal "[A] genuine identity sign can be flanked only by genuine singular terms",[22] so as this lapses for (2), it is false and the conclusion of the argument in which it figures can be avoided. To prevent confusions of the sort engendered by express ions of identity like (2) it would appear wiser to use a different sign for genuine identity.

But how does this restriction on singular terms restrict the values of x? By formally implementing a form of essential ism. For consider what the 'genuine/ non-genuine' distinction between names amounts to. If a rose by any other name is not a rose, then whatever the values of x satisfying (∃x) (x = the flowers growing in my garden) they do not include roses, for roses are not named by 'the flowers growing in my garden'. Similarly, with (4) x may range over objects named by genuine expressions like '2', '1 + 1' and '√4' which make M(3 > x) come out true, but not over objects named by 'the number of coins in my pocket'. It is this invidious attitude towards the different ways of specifying the values of x in these contexts that we call essentialism.[23] In particular, what Hirst appears to require, what we call FK essentialism, is a doctrine that provides a division on expressions or sentences or propositions into those that are essentially mathematical or ethical or aesthetic and so on, as opposed to those that are merely contingently so. For example, the objects satisfying (4) are FK essentially equal to 2, but only accidentally equal to the number of coins in my pocket at a certain time, and it is their essential properties which determine their suitability as values of x rather than their accidental properties.

The relation we are looking for to provide a basis for partitioning knowledge into forms is thus an equivalence relation defined on the FK essential properties of objects: for example Rj, ' ... has the same FK essential features as…'. Assuming we have at hand an essentialism that permits us to sort FK essential from contingent properties, Rj would also provide a basis for partitioning valid substitution instances required by the (substitutionally interpreted) quantification schemata for each form; for example contexts like (∃x)M(.....) for mathematics or (∃x) P(.....) for philosophy.

What makes the recognition of these assumptions so disastrous for Hirst is that from an epistemological point of view they impose an even greater burden on a theory of knowledge than the original 'forms' proposols. The business of. explicating the basis of what we have called FK essentialism, together with its attendant relation Rj appears far and away more exacting than the original task of finding some logical basis for partitioning knowledge into forms. What went wrong?

Recall that our only reason so far for even postulating essentialism was because we needed to account for an argument, namely (1) to (3) that challenged the intelligibility of talk about knowledge existing in forms. If we give up. talk of forms, the problems vanish. In (4) our criterion for sorting the required essential from merely contingent epistemic properties of objects traded on antecedent intuitive or traditional notions of what a mathematical form of knowledge was. Yet this was the very thing we were trying to explain. However, not only must Hirst produce some (epistemically) independent way of identifying expressions of FK essential, rather than accidental, mathematical knowledge, he also needs to be able to do it for every purported form, otherwise, because we can quantify across partition operators like M, boundaries will collapse in the manner of example (1) to (3). An understanding of how Rj applied to FK essential properties (or propositional objects) would, of course, solve this problem; but then this is where we came in.

What are the prospects for providing any logical basis for partitioning the OL elements of S in the manner required by the H-thesis? Is there any reason for thinking traditional groupings of knowledge reflect a logically necessary pattern, "an inescapable, fundamental, necessary organization"?[24] Our general worry, quite apart from the systematic question begging of Hirst's supporting claims, concerns the whole business of providing a suitable semantics not only for M but for any form operator on the OL elements of S. This is because for each proper subset of elements (corresponding to forms) of the set of knowledge claims S, we can construct an argument exactly parallel to one given by Quine[25] against necessary truth which shows that given a standard system of quantification with identity and definite descriptions, if one quantifies into members of the subset from outside, the distinction between set and subset collapses.[26] It is in this sense that we view knowledge as a 'seamless web' rather than a partitioned set. Our replies on behalf of Hirst to the difficulties involved in interpreting M carry over to this general argument as well, but as Hirst is concerned to defend forms without appeal to the sort of "metaphysical and epistemological realism"[27] he associates with the fully developed Greek notion of liberal education, setting foot on the primrose path that leads to essentialism is no doubt as unreasonable for him as it is for us. (We shall, nevertheless, explore more of this option on his behalf later on).

3. Derivability

So far we have focussed on reasons why various attempts to define a logical relation Ron S that will satisfy the H-thesis have been unsuccessful. But as the existence of an appropriate R is both necessary and sufficient for partitioning S, we can turn the problem around and say there must exist some R, whatever it may be if what Hirst has nominated as distinct forms can be shown to be disjoint. The obvious strategy here is to show that the forms are closed under derivability. Roughly, this means that, for example, ethical conclusions cannot be derived from non-ethical premises, or mathematical conclusions from non-mathematical premises. More formally we can say that for all xεP, where P is some form, if y is derived from x then yεP. To the obvious objection that our closure conditions presuppose an assignment of knowledge into forms, we can now retort that this assignment is made only after the derivability conditions have been satisfied or not. This means that closure of forms is something that results from applying the relation '…is derived from…' to elements of S.

At this point one may be tempted to ask why closure of forms under derivation in these circumstances is any different from partitioning S into disjoint subsets by an equivalence relation. The answer, naturally enough, is that there is no difference if elements of S are assigned to closed forms on the basis of derivability. Furthermore, as '…is derivable from…' is not symmetric, it will not do the job anyway. The result is that the obvious objection holds for relations that do not satisfy the H-thesis. In the absence of the sort of detailed studies to which Hirst has occasionally alluded,[28] the presumed closure of ethics, mathematics, religion and so on under derivability, far from providing an account of necessary divisions in knowledge, in fact presumes them, smuggled in as they are above, under the labels 'ethics', 'mathematics' and 'religion'.

4. Tests Against Experience

In view of the above difficulties with logical relations it seems to us that the most promising candidate for a partitioning relation that satisfies the H-thesis is Rt, '…is subject to the same kind of test against experience as…' Rt looks good because as well as being an equivalence relation it appears to specify a weak sense of 'sameness' that lies somewhere between the austerity of strict equivalence and the eclecticism of material equivalence. Can Rt satisfy the H-thesis? This will depend, in part, on whether (i) there is the same number of kinds of test against experience as there are forms of knowledge, and (ii) the different kinds of test against experience mark out different forms of knowledge.

To see how these matters can be approached, consider some x ε S, where x is subject to some kind of test T that is peculiar to form P, and there fo re x ε P. Is x ε P because T applies to x or does T apply to x because x ε P? If the former, then we need to supplement conditions (i) and (ii) with the claim that the distinctiveness of forms is constituted by the kinds of truth test used to assess the elements of S; that P is the form it is because of the nature of T. If this move is justified, the problem for Hirst simply shifts to producing some relation that partitions the set of tests into kinds that satisfy the above two conditions. Now if Hirst thinks that the meaning of a sentence or proposition can be identified simply with its truth tests then the task of producing this relation is equivalent to our earlier one of producing an R to satisfy the H-thesis. If there is some important difference the task is more complex, as specifying differences in kinds of test must now proceed without recourse, even in principle, to a complete understanding of the elements of S. It is prob ably for reasons like these that Hirst appears more sympathetic to the latter alternative of viewing the appropriateness of a test on x as a function of the form to which x belongs.[29] However, as a device for explaining the basis on which knowledge is partitioned into forms, an appeal to Rt now becomes otiose as our understanding of 'tests' in the required sense is predicated on a prior understanding of the very 'knowledge as forms' claim Rt was supposed to explain.

Granted these difficulties, can the job be done at all? That is given the resources available in good epistemology, can we produce a clear account of a partitioning relation that is based on some account of tests against experience? We think not, and for reasons parallel to the sorts of argument we mounted against the possibility of there being a logical relation that would satisfy the H-thesis.[30] Space prevents us from pursuing the matter further here. Finally, however, we would like to attempt - in no more than a sketch - to state a theory which can explain (i) the fruitlessness of Hirst's search for partitioning logical, semantical and evidential relations, and (ii) the historical genesis, and some of the consequences of the forms thesis for the epistemic enterprise. We should stress, of course, that our explanatory theory can be stated independently of our above minimum case argument against the forms thesis, and vice-versa, so that the above arguments can stand on their own merits. On the other hand, although our statement of the theory in this paper is sketchy, our above minimum case argument against the forms thesis functions, at several points, as an argument for our alternative materialist epistemology, which is the alternative theory we are advocating. The main thrust of the theory is to expose some errors about necessity, and the effects of these errors on the social relations of the quest for knowledge.

5. Materialism

Our theory asserts, first, that the basic units of knowledge are theories, not forms.[31] While we admit the possibility of non-sentential expressions of a given theory, we may characterize a theory as a fabric of deductively ordered declarative sentences, expressed in a given language, where the language may be but need not be peculiar to the theory, and where the inferential sequences include or entail hypotheses about the way the world is, works, changes, etc. Theories in non-sentential form (e.g. in the visual and plastic art forms) can then be identified given rules of translation into sentences, and vice-versa, where the theory of translation, with appropriate analytic hypotheses, rather than essential entities such as propositions or meanings, carries the semantic burden.

On this account, the meaning of a term is determined largely by its relation to other terms within the same theory, arid so the same word, e.g. 'education' or 'knowledge', within the same language, e.g. English, can have different meanings, depending on which theory is governing its use.

Theories may or may not be in competition, i.e. inconsistent with one another. Theory T_1 is in competition with T_2 when one or more of the sentences of T_1 is contrary to sentence(s) of T_2.[32] For this situation to obtain, T_1 and T_2 must be addressed to the solution of at least one common problem.[33][34]

The existence of theories is to be explained causally, and therefore materially, as problem-solving procedures. Theories are not only to be read, but to be judged and assessed as solutions to practical problems (including, especially in the case of epistemology, the problems of theoretical practice). Clearly, there are certain issues concerning whether a theory is addressing the right problem, and we would need (a theory of how) to distinguish between real problems and pseudo-problems, and between better and worse formulations of problems. A large part of our machinery here would lean on a theory of evidence and experiment, on the material relations between theory and practice.

In explaining and assessing theory-competition in this way we note both the theory-ladenness of observation and experimental practice generally, and the existence of what Lakatos has felicitiously called "touchstone theory".[35] Granted the theory-ladenness thesis, we require an explanation as to how the theory-laden observation can play a part in choosing between rival theories. For sufficiently wide ranging accounts of the way the world is, the observational evidence is not in dispute, nor is a certain modicum of logic, mathematics and semantics. These can therefore be used by one theory against another if the language in which these touchstone statements are expressed is shared by these rival theories. By way of contrast to the forms of knowledge thesis, with its stress on the logical necessity of various procedures for tests against experience, touchstone is not made up of epistemically favoured statements. It is merely the shifting and historically explicable amount of theory that is shared by rival theories and/or theorists.[36]

We are now in a position to provide an account, in competition with Hirst's for the apparent existence of different areas of knowledge. (Let us call them areas to avoid confusion with Hirst's forms and fields). Theories T_1, T_2, and T_3 may be in very close competiton, reflected in considerable overlap of problems addressed, and a specific block of touchstone. The nearer the inter-theoretic formulation of problems and the more stable and effective the touchstone, the more inclined are we to recognise a clearly defined area of knowledge. Nevertheless, 13 may also be in more distant competition with T_4, where T_4 is in close competition with T_5 and T_6. An example is the psychology/social psychology/ sociology configuration of theories. It will be noted that the distinctions between areas may be loose and blurred. The borders are, of course, subject to change. Discovery of common problems between areas of knowledge occasionally shifts boundaries, or creates new areas, e.g. biochemistry. Moreover, if we may critically assess problem selection and formulation, it follows that distinctions between areas may be unhelpful, or even wrong, insofar as they prevent the acquisition of further knowledge. The acknowledgement that areas may be better or worse constituted entails no more than a recognition of the historical conditions for progressive theory development, not any suspicion that there might be essentially derived form-specific criteria better or worse observed in practice.

It thus becomes otiose, in explaining the historical development of subjects, or areas of knowledge, to postulate a set of logically necessary epistemic properties which have gradually been discovered over the centuries. It becomes easier to explain the changes in organization and boundaries of areas, and to recognise ways in which they can degenerate, as well as progress. Prevailing modes of problem formulation may receive severe jolts which have a shake-out effect on the current set of areas. Consider the impact of Newtonian mechanics and Darwinian biology, and the drastic effects of each upon problem formulation in theology.

So far as both theory rivalry and touchstone are concerned, logic and mathematics are areas containing theories on a par with any others, from an epistemological point of view,[37] albeit that they are theories of the highest levels of generality and abstraction. If we may be permitted a psychological speculation, the generality and apparent solidity of the contents of these areas may have misled philosophers such as Hirst into essentialist views of logical necessity. But logic, mathematics, and some semantics, though not without their controversial aspects, are no more than the most impressive blocks of generally applicable touchstone yet produced. Touchstone in the physical sciences comes close, but as yet is mostly confined to the physical sciences. Although we have limited sympathy for positivism,[38] it seems to us that physical theory touchstone could usefully find some wider application in the social sciences.[39] Another way of putting this point is to suggest that we presently have some bad area grouping.

This account of the growth of knowledge as theory competition in view of practical problems and touchstone enables us to make some critical and explanatory remarks about the forms of knowledge thesis. In criticism, we can point out the error of relying (notwithstanding Hirst's disclaimer) on the generally applicable concepts or logical rules of one putative form (that of which logic is an element) to provide the basis for specifying the partitioning relations between it and all t he others. Next, we can explain the collapse of partitions by reference to our alternative account of logic as touchstone, which escapes the Hirstian dilemmas of form-specific concepts or logical rules versus generally applicable concepts or logical rules, of independent versus totally independent forms, and of a meta-language which on the one had has to be logically distinct from the object language but on the other hand itself becomes an object language within the theoretical framework of the H-thesis.

Instead of having to confront such irresolvable dilemmas, we can point out that the rational function of expressions such as 'Ɔ', ' ≡ ', ' ≣ ', ' → ', '↔ ',' ⇒', and '⇔' lies in their deployment in the facilitation of theory development and competition, thus abetting the growth of knowledge.[40] These predicates, therefore, are valuable only on functional, pragmatic grounds, and not because they unlock for us any essential properties of the world, or of our knowledge of it. In this respect touchstone, in the final analysis, like any other theory, is accounted for as a form of problem-solving social practice.[41] To cast such predicates in any other role is to generate considerable and useless theoretical baggage. It is also, as we have said, to become committed to mistaken views about necessity, in particular the view that there are certain necessary properties. Now while, for better or worse, there are several senses in which the word 'necessity' is currently used,[42] for our present purposes we shall distinguish between the Hirstian use of 'necessity' to refer to (logically) necessary properties, and what we shall call hypothetical necessity. The latter obtains where one sentence in an argument follows deductively validly from some conjunction of preceding sentences. The utility of hypothetical necessity depends on it being true that, within the framework provided by some theory or conjunction of theories, we can construct valid arguments. Tins necessity is a semantical predicate attaching to names of statements, such as 'snow is white' in the argument 'if grass is green then snow is white; grass is green; therefore snow is white'. The predicate 'hypothetically necessary' thus applies univocally to deductively validly inferred statements within any theory. Semantically, 'hypothetical necessity' is equivalent to 'logically valid'. '9 > 5' is necessary in this sense within the framework of mathematics. Demonstrating that it is would oblige us to back up into the axioms or primitives of mathematics. Any observation on such a demonstration would constitute a metalinguistic comment on a sequence of statements in the object language, in this case the language of mathematics. 'Hypothetical necessity' would be used as a predicate in ML, a term used to describe deductive argument within theories. Other than 'hypothetical necessity', we would maintain, no necessities are conferred by logic, which, therefore, does not prop up necessary epistemic or on to logical properties.

If there are no such necessary properties, then any theory declaring their existence is not merely false, but systematically distorts any epistemological understanding of our capacities for knowledge - i.e. progressive theory development We may say that such a theory necessitizes a regressive set of epistemic procedures and social relations of theory production and, we should add, of necessitizes, by proclaiming them as logically necessary, the social relations of a certain division of theoretical labour.

Taken further, this argument would lead into a theory of ideology, in which a central claim would be that ideology, and therefore ideological epistemologies, retard our understanding of the world, and therefore and to that extent prevent us from changing it (solving our problems within it). The blockage exists simultaneously in theory and practice, since the process of experiment, a process demanding conceptualization and rational execution, is part of the process of progressive theory development.[43]

We would then need to explain why such ideological theories get a grip on people's minds and become embedded in their practice. We would approach this problem by examining, in the first place, the social relations of the production of ideological theories. In the case of the forms thesis, this would involve looking at the evolution of the practices of theory production in analytic philosophy of education,[44] and locating then, within, both, wider developments in philosophy, and also the nexus between the sites of theory production for the teacher training industry on the one hand and the practices of institutionalised education on the other. We would claim that the forms thesis functions to necessitize, by spurious appeals to logic, a regressive set of social relations for the production and transmission of knowledge, in which certain professional, bureaucratic and political interests are socially legitimated by a tendentious epistemology disguised as a thesis in philosophical logic.

We have, however, neither the desire nor the space to press such claims here. We conclude with the comment that what has just been outlined suggests a social-epistemic analogue of the epistemological thesis of the unity of knowledge. The practical problem becomes one of so organizing the division of theoretical labour so as to eliminate the institutionalised and ideologically represented and legitimated schisms which arrest the achievement of knowledge, and thus its unification. Instead, as represented by a monistic materialism, tasks would be organized so as to maximise theoretical coherence and practical epistemic power. Logic becomes at one with the experimental venture of intervention in the causal Process of natural and human history.

Notes and references

1. In Hirst, P. H., Knowledge and the Curriculum, Routledge and Kegan Paul, 1974, 87.
2. Ibid., 66.
3. Hirst, P. H., "Liberal Education and the Nature of Knowledge", ibid., 38.
4. Hirst, P. H., "Language and Thought", ibid., 76.
5. For the distinction between substitution and value we rely on Quine. See, for example, Quine, W.V.O., "Reply to Professor Marcus", in his The Ways of Paradox, Harvard University Press, Cambridge Massachusetts, 1976, 178-9, and "Logic and the Reification of Universals", in his From a Logical Point of View, Harper and Row, New York, 1963, 107-112.
6. For more on use and mention see, for example, Quine, W.V.O., Methods of Logic, Routledge and Kegan Paul, London, 37-8.
7. To take in Hirst's account of art as expressing statements, in "Literature and the Fine Arts as a Unique Form of Knowledge", in Hirst, op. cit., ch. 10, we would have to correspondingly expand our view of 'sentence'.
8. For arguments in favour of dispensing with propositional objects, see Quine, Word and Object, M.I.T. Press, Cambridge Massachussetts, 1960, 206-211, and Quine, Philosophy of Logic, Prentice-Hall, Englewood Cliffs, 1970, Ch.1.
9. Hirst, 11 Liberal Education and the Nature of Knowledge" in Hirst, op. cit., 38.
10. Hirst, "The Forms of Knowledge Revisited", ibid., 89.
11. Loc. cit.
12. The proof that an equivalence relation is both necessary and sufficient for partitioning a set into disjoint subsets is standard, and can be found in most modern algebra textbooks dealing with set theory. We have made use of Paley, 1-1., and Weichsel, P.M., A First Course in Abstract Algebra, Holt, Rinehart and Winston, New York, 1964, 10-22, and Whitesitt, J. E., Principles of Modern Algebra, Addison-Wesley, 1964, 23-26. (a) We prove the sufficiency part first, namely, that if R is an equivalence relation defined on a set S, it partitions S into disjoint subsets. Proof: Call the set of elements equivalent to a given element 'a', 'the equivalence class determined by a', and denote this by [a] where

    [a]= (x:x ε S & xRa).

    We first need to prove the following lemma.

    Lemma: If aRb then [a]= [b].

    Proof: Let aRb and let X ε [a]. Then by definition xRa. But since R is transitive, xRb. Therefore, [a] $\subseteq$ [b]. Now since R is symmetric bRa, and by a similar argument interchanging a and b we get [b] $\subseteq$ [a].

    Therefore [a]= [b].

    Using this lemma we can now prove that for a, b ε S either [a] n [b]= Ø or [a] = [b].

    Proof: Either [a] n [b] = 0 or [a] n [b] I IL If the former then the subsets of S are disjoint. If the latter, then there exists an element c ε S, such that c ε [a] and c ε [b]. Therefore cRa and cRb. But since R is symmetric aRc, and since R is transitive aRb. But according to our lemma if aRb then [a] = [b] and so [a] n (b] = Ø. Moreover, as each element of S is found in one and only one subset of S, Sis equal to the union of all the [a].

    (b) We prove the necessity part next.

    Proof: Let S be a set, S ≠ Ø and P a collection of subsets (X1, X2, X3, … , Xi, …) such that (1) the Xi's are disjoint and (2) the union of all the Xi's is S. Let R be any relation on S such that aRb if a and b both belong to the same Xi. Then (i) aRa since a is in one and only one Xi. (ii) If aRb then a and bare the same Xi, and so bRa. (iii) If aRb, i.e. a and bare in the same Xi, then a and care in the same Xi and so aRc. Therefore R is an equivalence relation.

13. Watt, A. J., "Forms of Knowledge and Norms of Rationality", Educational Philosophy and Theory, Vol. 6, No. 1, March 1974, 10.
14. The importance of an independent specification is something we shall harp upon throughout this paper. Its importance is emphasised by Phillips, D.C. in "Features of Forms of Knowledge", Educational Philosophy and Theory, Vol. 3, No. 2, October 1971, p. 29, where he says "[S]omewhere the circle in which he [Hirst] is moving needs to be broken; he needs to establish and not merely to assert that the four distinguishing features actually do distinguish, and he needs to establish that the seven 1 forms of knowledge' which he identifies actually do differ with respect to these four 'distinguishing features'."
15. A good discussion of these issues may be found in Quine, Mathematical Logic, Harvard University Press, Cambridge Massachusetts, 1941, 23-37.
16. Hirst, "What is Teaching?" in Hirst, op. cit., 106.
17. A good informal account of the semantics for these modal notions can be found in Hughes, G. E., and Cresswell, M. J., An Introduction to Modal Logic, Methuen, London, 1972, Ch.2.
18. For a critical discussion of conceptual analysis, see Evers, C. W., "Analytic Philosophy of Education: from a Logical Point of View", Educational Philosophy and Theory, Vol. 11, No. 2, November 1979, 1-15.
19. A brief, general discussion of referential opacity may be found in Quine, Word and Object, M.I.T. Press, Massachusetts, 1960, sections 30, 31 and 32. Its relation to the modal notion of logical necessity is explored in Quine "Reference and Modality", in Linsky, L. (ed.), Reference and Modality, 'Oxford University Press, London, 1971, 17-34, and a brief discussion, from a Quinean perspective, of opacity in some accounts of 'conceptual truth' in philosophy of education may be found in Evers, C. W., op. cit. The best comprehensive treatment is still Follesdal, D., Referential Opacity and Modal Logic, Universitetsforlaget, 1966.
20. This form of wording together with a discussion of objectual and substitutional quantification are in Quine, "Reply to Professor Marcus", in The Ways of Paradox, op. cit., 182.
21. See, for example, Follesdal, D., "Quantification into Causal Contexts", in Linsky, L., op. cit., 52-62, and "Knowledge, Identity and Existence", Theoria, Vol. 33, Part 1, 1967, 1-27.
22. Follesdal, D. "Quantification into Causal Contexts", op. cit., 60. A defence of this move is also in Baldwin, Thomas, "Quantification, Modality and Indirect Speech", in Blackburn, Simon, (ed.), Meaning, Reference and Necessity, Cambridge University Press, Cambridge, 1975, 76-83.
23. Following Quine, "Reference and Modality", op. cit., 30.
24. Hirst, "Curriculum Integration", in Hirst, op. cit."", 135.
25. Quine, Word and Object, op. cit., 197-199.
26. This general formulation is due to Follesdal in his "Quine on Modality", in Davidson, D., and Hintikka, J., (eds.), Words and Objections, D. Reidel Publishing Co., Dordrecht, 1969, 179.
27. Hirst, "Liberal Education and the Nature of Knowledge", op. cit., 32.
28. For example, Hirst, "The Forms of Knowledge Revisited", op. cit., p. 85, and Hirst, P. H. and Peters, R. S., The Logic of Education, Routledge and Kegan Paul, London, 1970, 63.
29. We say "appears" because, matters of emphasis aside, Hirst displays a reluctance to make a judgment on this issue. See his "Realms of Meaning and Forms of Knowledge" in Hirst, op. cit., 60-61.
30. Our strategy in demonstrating such parallel reasons would be to try to show that any attempt on behalf of the forms thesis to specify a relation which holds between knowledge and the object of knowledge leads to a contradiction, in that the forms intersect viciously with one another along the axis of the relation.
31. See, e.g., Quine, W. V., "Two Dogmas of Empiricism", in From a Logical Point of View, Harvard University Press, Cambridge Massachusetts, 1961 and Harding, S. G., Can Theories be Refuted? D. Reidel, Dordrecht, 1976.

32. Following Lakatos, I., "Falsification and the Methodology of Scientific Research Programmes", in Lakatos, I., and Musgrave, A., (eds.), Criticism and the Growth of Knowledge, Cambridge, Cambridge University Press, 1970, 91-196.

33. Popper has contributed much to our understanding of theories as problem solving procedures. See, e.g. Popper, K. R., "Epistemology Without a Knowing Subject", Objective Knowledge, Oxford, Oxford University Press, 1972, p. 119. (Of course we must reject his "Three Worlds" thesis). But our sketch here of an epistemology utilising the problems/solutions schema is deeply indebted to some as yet unpublished work by John Burnheim of Sydney University. We doubt, however, that Burnheim would concur with all aspects of our discussion.

34. For instance, Piaget's theory competes with Skinner's in that they both address the question of how people learn, and are incompatible with each other in certain respects. However, neither is in conflict with Einstein's general theory of relativity. They are both in conflict with M.F.D. Young's theories of knowledge and its acquisition, though not in nearly such c lose competition with that as with each other, since Young's theories are concerned with rather a different cluster of problems, only some of which concern Piaget and Skinner. For a popular account of some of these points, see Walker, J. C., and Evers, C. W, "Epistemology and Justifying the Curriculum of Educational Studies", British Journal of Educational Studies, Vol. 30, No. 2, 1982, 213-229.

35. Lakatos, loc. cit. While we think that Lakatos is basically right about the importance of touchstone and the kind of role it plays, we do not think that he has provided an adequate account of the way in which touchstone can, or could work. In short, Lakatos, like Popper, lacks the materialism which we hold can provide such an account.

36. We say theory and/or theorists simply to make the point that in the las t analysis theorists decide what they will accept as touchstone, and that this may or may not in each particular case coincide with procedures suitable for or compatible with the respective rival theories.

37. We recognise that this is a controversial account of logic and mathematics.

38. Especially in respect of positivist doctrines of meaning, such as logical positivist verificationism, where semantics is used to confer unwarranted epistemic privilege on certain theories. (An error widely reproduced in analytic philosophy of education).

39. Probably the clearest example is the case of econometrics, which forces the exact methods (e.g. certain mathematical methods) and some of the concepts of the physical sciences onto fairly recalcitrant material.

40. Indeed, we hold that logic is touchstone to the setting up of any theory or theory competition at all.

41. Although we have noted the controversiality of this view of logic, here we are confining our remarks to urging its critical and explanatory superiority over and in relation to accounts such as Hirst's. For some relevant discussion see Nerlich, G. C., "Pragmatically Necessary Statements", Nous, Vol. 7, 1973, 247-268; and Devitt, M., "Against Incommensurability", Australian Journal of Philosophy, Vol. 57, No. 1, 1979, 29-50.

42. See Walker, J. C., Autonomy, Authority and Antagonism: A Critique of Liberal Rationalist Ideology in Philosophy of Education, unpublished Ph. D., thesis, Department of Genera Philosophy, University of Sydney, pp. 244-254, shortly to be presented in a paper, "'Necessity', Necessitism and Ideology".

43. It is in this sense that we would interpret, and endorse, Marx's statement, "The philosophers have only interpreted the world in various ways; the point, however, is to change it." (Eleventh Thesis on Feuerbach.) Interpreting the world progressively involves experimental intervention in the world, a causal process involving changes in the world.

44. See Walker, J. C., "The Evolution of Analytic Philosophy of Education", paper presented at the Annual Conference of PESA", Brisbane, August 1979.