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RESPONSE

Constructivism without epistemology

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Olssen defines 'constructivism' initially, "as a movement with its origins in developmental psychology, originating with the work of Jean Piaget and receiving its contemporary expression in the radical constructivism of von Glasersfeld and others." Later he adds to the definition, "a sceptical position in relation to epistemology". He concludes that this "leads to idealism and invariably to relativism, as it is impossible to say that one view of the world or of the state of the society is any more valid than any other". Like Olssen I believe that this radical version of constructivism is not epistemologically tenable. I also believe that in its educational applications it is at best silly and at worst dangerous. This is, however, but one version of constructivism amongst, it is claimed, nineteen possible versions (noted in Nola, this volume). Glasersfeld's version is, however, a particularly pernicious form.

As there is much with which I agree in Olssen's paper, rather than dealing with his critique in detail, I wish to extend the debate on constructivism, looking at a version of constructivism to be found in the writings on mathematics of Ludwig Wittgenstein. I will argue that this version of constructivism does not reproduce the idealistic, relativistic and sceptical traits of radical constructivism but, instead, can be seen as non-idealistic and objective, and as supporting correct procedures in mathematics (without relegating the teacher to a facilitator). Yet it is sceptical of the search for epistemological foundations for mathematics.

In more than one source Wittgenstein suggests that the mathematician is more of an inventor than a discoverer. For example (LFM, I: 22):

> I shall try again and again to show that what is called a mathematical discovery had much better be called an invention.

Philosophical commentators on Wittgenstein seem to interpret him as being on the side of mathematicians as inventors - for example Kenny (1973: 18), referring to Wittgenstein's work in the early 1930's on mathematics, says:

> According to realist philosophers, the mathematician is a discoverer; according to constructivist philosophers, he is a creator. Wittgenstein now sided very definitely with the constructivists. 'History can be made or written' he said to Waissman; 'mathematics can only be made.'

Kenny is not alone in saying such things about Wittgenstein's approach to mathematics (eg., Klenk, 1976: 2 - "the mathematician is an inventor, not a discoverer"). Even if Wittgenstein sides with the inventors, and mathematics is seen by him as a social construct, it will be argued it is in a special sense, and not one which lends support in mathematics education to an approach which can be called constructivist in which learners are said to construct mathematical knowledge for themselves. First I will discuss his criticisms of idealism, mental states and constructions. Then I will explain how, whilst still seeing mathematics as constructed by human beings he is not sceptical about the objectivity of mathematics. For Wittgenstein not anything at all could be constructed as an outcome of a mathematical procedure.

Wittgenstein attempted to bypass the epistemological arguments about the foundations of mathematics, by dissolving them in the same manner as he had dissolved philosophical problems associated with language, by explication of the logical structures of mathematical propositions. Asking epistemological questions about the foundations of mathematics involved, then, an idle use of language for Wittgenstein, attempting to ground in firm foundations something that was not in doubt.

Anti-idealism

Just as Wittgenstein rejects the logicist notion that mathematics is to be grounded by reference to abstract mathematical objects, so also does he reject the view that mathematics is to be justified by reference to mental entities, as in his attack upon intuitionism. Intuitionists believe that mathematics is a study of certain kinds of *mental* entities or objects which are used in mental constructions. These mental constructions become then the subject of a form of mental contemplation. What a mathematical theorem says is that a certain mental construction has been effected and to understand mathematics is to have through contemplation, some inner grasp, knowledge, or *intuition* of the objects that are the building blocks of these constructions.

The only thing that really matters in mathematics for the intuitionists is the inner mental process. Indeed the outer linguistic representations (spoken or written) may be considered as incidental, or as serving an auxiliary function, or even as being potentially ambiguous and misleading. Since we cannot ever be certain that linguistic expressions are correct representations and translations of our inner mental constructions, mathematical language cannot be trusted.

In intuitionism, then, it is not mathematically important that we know how to *use* or apply a mathematical proposition. Being able to use a mathematical proposition such as '2 + 2 = 4' is not a criterion for understanding that proposition. Similarly to make an inference in mathematics is not to write down one proposition followed by another but, rather, to perceive internally connections between the two propositions. Correctness in mathematics is not, then, given by observing spoken or written use of mathematical propositions and comparing them with the beliefs and behaviour of others who use and apply mathematics but instead they are defined in terms of inner mental states.

Essentially, then, we are presented with a mental image theory of understanding. Mental images are both necessary and sufficient for understanding mathematics so that when we claim to understand a mathematical proposition we are in possession of a unique mental state. Being able to apply and use a mathematical proposition is not necessary for the correctness of any claim that we understand that proposition.

Intuitionists clearly reject any views that mathematics is a form of discovery about pre-existing and mind independent objects. Due to the emphasis upon mental states, mental constructions, and a denial of reality in mathematics to anything external to the mind, they seem to be close to some beliefs of constructivists.

Wittgenstein has much to say about the mental image account of understanding (eg, PI, 13 8-184; RPP, Vol.II: 63-100). He attacks the intuitionists in the prioritising of mental states over the linguistic accompaniments of those internal states. Wittgenstein stresses that it is the use of the mathematical language that is important in mathematics. The notion of an internal mental state is inadequate for an account of understanding. In mathematics understanding is given by the use of mathematical statements. To understand a mathematical proposition, then, is to understand a language and to master the use of that language (PI, 199). The occurrence of a mental image without an application or use cannot provide understanding in mathematics.

Wittgenstein's arguments in PI address not only the sufficiency of mental images for understanding in mathematics but also their necessity (PI, 141-142). Wittgenstein stressed also the objectivity of mathematics and mathematical inference. For him inference is writing down one proposition after another according to tried practice and is not dependent upon a particular mental state.

Given the centrality of mental constructions for the radical constructivist's account of mathematics it is clear that Wittgenstein can have little in common with them.

Objectivity

Wittgenstein was not interested in the foundations of mathematics, and at several points derided Russell's and Frege's logicist approaches to the foundations of mathematics. This anti-foundational approach is quite clearly stated (RFM, V13):

> What does mathematics need a foundation for? It no more needs one, I believe, than propositions about physical objects - or about sense impressions, need an analysis. What mathematical propositions stand in need of is a clarification of their grammar, just as do those other propositions. ... The mathematical problems of what is called foundations are no more the foundations for us than the painted rock is the support of a painted tower.

Foundational objectivism in mathematics is a belief that theories have as their subject matter a set or sets of abstract mathematical objects. Numbers, real numbers, groups etc. are understood as abstract but existing objects referred to by the terms of the theories. These abstract objects exist independently of human minds. Hence, mathematics becomes the *discovery* of the relations and properties of these abstract entities, and mathematical statements become true in so far as they correspond with the mathematical reality. Mathematical statements tell us something about the reality of these entities. They are not linguistic entities which are related logically to other linguistic entities in mathematical structures.

Underlying this view is the more basic thesis that number words operate like physical object words - that they are *referential.* This may be to commit a fundamental mistake for not all terms in our language refer to entities or denote entities. If not all terms denote then perhaps not all statements are factual. The abandonment of this position on meaning as being referential is a crucial turning point for Wittgenstein (PG, Part II).

What is at issue, then, is the function of mathematical language. Central claims in Wittgenstein's position on mathematics are that there are no mathematical objects in this sense and that mathematical propositions are *not* statements of fact, nor propositions about entities of any kind. Mathematical propositions are assigned a particular function or role in language which is not merely different from, but is in *conflict* with, that factual function which certain propositions fulfil in language.

For Wittgenstein the logical grammar of mathematics is *normative.* Rather than describing matters of fact mathematical propositions provide a normative but not merely linguistic structure in which we can infer propositions. This structure is itself constructed and truth depends upon correct derivation, where correct depends upon standard usage of terms and correct inference procedures which have themselves been derived from experience. That we must place 5 after 4 in the number series is a normative 'must' required by the rule following character of mathematics. The truth is not guaranteed by abstract entities or logical necessity or even linguistic convention, but by the rules of logical syntax of mathematical propositions, and these rules are normative in nature.

For Formalists in the foundations of mathematics it is the external linguistic representations, the signs themselves, that constitute the subject matter of mathematics. Formalists reject the views of intuitionists. Manipulation of symbols according to the formal axioms and rules of the system reduces mathematics to a game played with meaningless symbols. The only meaning which the symbols seem to have, apart from finitary arithmetic, is that given to them implicitly by the axioms and rules of the formal system.

Wittgenstein firmly rejects the view that mathematical signs are meaningless and that inference is a mere game of logical manipulation of these symbols. Then mathematics would have no need for human agents as it could be a purely mechanised system. This would divest it of the notion of a language, the metaphysics of a form of life, and the normative features of mathematical propositions so central to Wittgenstein's philosophy.

Whilst Wittgenstein rejects the notion that abstract mathematical objects can serve as a guarantor of mathematics, and that objectivity is guaranteed logically by a formal system, he does not reject the notion that mathematics is objective. Indeed he is at pains to argue that mathematics is an area where personal preference and interpretation can play no role. This objectivity is to be found in human practice in which options may be deeply submerged and not at all obvious, and in the normativity of mathematical propositions.

Wittgenstein was not engaged in a sceptical assault upon the foundations of mathematics but rather he was arguing that the principal problems in what passed for the foundations of mathematics stem from conceptual confusion. Nor was he attempting to provide a reconciliation between the competing theoretical approaches to the foundations of mathematics nor, indeed, advance a new and competing theory. He was raising conceptual questions about the very framework in which these competing theories had been developed. He was arguing that, instead of searching for epistemological foundations for fundamental propositions which cannot be doubted, the logical syntax of such propositions must be clarified. Problems in the foundations of mathematics were to be resolved, then, by dissolving them, getting rid of the framework and the epistemological questions themselves, with the same techniques that he used in the philosophy of language (Shanker, 1987: 28). Mathematics is not descriptive but normative, prescribing to us how to proceed correctively to arrive at correct conclusions, from the logical structures of mathematical propositions. In talking of proof Wittgenstein illustrates this well (RFM, #63)

> I *go through* the proof and then accept its result. - I mean: this is simply what we *do.* This is use and custom among us, or a fact of our natural history.

For Wittgenstein, it is important to describe the role that mathematical propositions play in human practices and our ordinary everyday language, so as to resolve puzzles that appear to arise. Philosophy leaves everything as it is. According to Wittgenstein philosophy can only describe the important normative role that mathematical language and practices have for us as human beings: one cannot explain these by appeal to underlying real mathematical entities, or to some form of intuition or formalist theory, or to empirical facts. Thus he parts company not only with the logicians but also with the intuitionists, the formalists, and the empiricists.

For Wittgenstein the puzzles which arise in the foundations of mathematics are similar to philosophical puzzles in general and are embedded in language and its misuse (LFM, IV: 44).

> I am trying to conduct you on tours in a certain country. I will try to show you that the philosophical difficulties which arise in mathematics as elsewhere arise because we find ourselves in a strange town and do not know our way. So we must learn the topography by going from one place in the town to another, and from there to another, and so 011. And one must do this so often that one knows one's way about, either immediately or pretty soon after looking around a bit, wherever one may be set down... The difficulty of philosophy is to find one's way about. The real problem in philosophy is a matter of memory - memory of a peculiar sort.

He embeds mathematical propositions firmly in his notion of language games, as mathematical propositions bear resemblances to propositions in other language games in a complicated network of overlapping and cris-crossing similarities.

> I can think of no better expression to characterise these similarities than 'family resemblances'; for the various resemblances between members of a family: build, features, colour of eyes, gait, temperament, etc. etc. overlap and crisscross in the same way. And I shall say: 'games' form a family.

> And for instance the kinds of number form a family in the same way. Why do we call something a 'number'? Well, perhaps because it has a *direct* relationship with several things that have hitherto been called number; and this can be said to give it an indirect relationship to other things we call the same name. And we extend our concept of number as in spinning a thread we twist fibre on fibre. And the strength of the thread does not reside in the fact that some one fibre runs through its whole length, but in the overlapping of many fibres. (PI, 167)

In summary, then, we can say of Wittgenstein that he rejects the traditional foundational approaches to mathematics and instead introduces a robust naturalistic and normative 'foundation' where mathematics is part of a form of life and a language game. Not only did Wittgenstein argue that there were no foundations to mathematics in the Russell/Frege sense and that logic did not explain mathematics but also that the very notion of foundations was one of the traditional philosophical puzzles that arise when language goes on holiday. Instead he makes the point quite firmly that children do get to the 'foundations' of mathematics (LFM, XXVIII: 27; OC, 47)

> a child has got to the bottom of arithmetic in knowing how to apply numbers, and that's all there is to it.

In order to do this we have to teach, particularly techniques (LFM, VIII: 83)

> There is no discovery that 13 follows 12. That's our technique - we *fix*, we teach, our technique that way. If there is a discovery - it is that this is a valuable thing to do.

Wittgenstein shows us that mathematics is deeply embedded in human practices and to that extent is a type of human construction, but there are no personal constructions that would 'permit' 15 follows 13 in the number series. He construes mathematics under the more general rubric of language games which are deeply embedded in forms of life. Whilst being far from sceptical about the objectivity of mathematics he nevertheless maintains that epistemological searches for foundations in mathematics are misplaced. Mathematics is objective because the language game prescribes to us how to proceed correctly. Mathematics has no epistemic foundations in individualistic mental states and personal constructions, as mathematical understanding can only be shown in correct practice. As Nola argues (in this volume) understanding is an achievement word, the meaning of which depends upon its correlate of misunderstanding. Radical constructivism is quiet on what counts as correct and incorrect mathematical understanding. Wittgenstein is not.

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